**ФМП\***

1. Доказать, что данная функция удовлетворяет данному уравнению.
2. Найти производную по направлению и градиент функции в точке $M\_{1}$.
3. Вычислить приближенно с помощью дифференциала.
4. Найти уравнение касательной прямой и нормальной плоскости к линии L в точке M0.
5. Найти локальный экстремум ФМП.
6. Найти условный экстремум ФМП.

**Вариант 21\***

$$21.1. z=sin^{2}\left(x-2y\right); 4z\_{xx}^{''}=z\_{yy}^{''};$$

$$21.2. z=ln\left(x^{3}+y^{2}+z+1\right);$$

$$ M\_{1}\left(1;3;0\right); M\_{2}\left(-4;1;3\right);$$

$$21.3. \sqrt[3]{3,61^{2}-0,95^{2}};$$

$$21.4. \left\{\begin{array}{c}x^{2}+y^{2}=2\sqrt{2}x\\z=x^{2}+y^{2}-4\end{array}\right.; M\_{0}\left(-\sqrt{2};-\sqrt{2};0\right);$$

$$21.5. z=x^{3}+y^{2}-6xy-39x+18y+20;$$

$$21.6. z=x^{2}+y^{2}; при x+y-2=0;$$

**Вариант 22\***

$$22.1. z=\frac{xy}{x+y}; x∙z\_{x}^{'}+y∙z\_{y}^{'}=z;$$

$$22.2. z=e^{xy+z^{3}}; M\_{1}\left(-5;0;2\right); M\_{2}\left(2;4;-3\right);$$

$$22.3. 2,003^{2}∙3,998^{3}∙1,002^{2};$$

$$22.4. \left\{\begin{array}{c}x^{2}+y^{2}=4y\\z=6-x^{2}\end{array}\right.; M\_{0}\left(-\sqrt{3};1;3\right);$$

$$22.5. u=x^{3}+y^{2}+xy-2zx+3y+2z^{2}-1;$$

$$22.6. z=x^{2}+2y^{2}; при 3x+2y-11=0;$$

**Вариант 23\***

$$23.1. z=ln\left(x+e^{-y}\right); z\_{x}^{'}∙z\_{xy}^{''}-z\_{y}^{'}∙z\_{xx}^{''}=0;$$

$$23.2. z=\left(x^{2}-y^{2}+z\right)^{3}; M\_{1}\left(1;-1;2\right); M\_{2}\left(0;-1;3\right);$$

$$23.3. 1,002∙2,003^{2}∙3,004^{3};$$

$$23.4. \left\{\begin{array}{c}x^{2}+y^{2}+z^{2}=16\\x^{2}+y^{2}=4\end{array}\right.; M\_{0}\left(2;0;-2\sqrt{3}\right);$$

$$23.5 z=x\sqrt{y}-2x^{2}-y+14x-1;$$

$$23.6. z=\frac{1}{x}+\frac{1}{y} при x+y-6=0;$$

**Вариант 24\***

$$24.1. z=arctg \frac{y}{x}; z\_{xx}^{''}+z\_{yy}^{''}=0;$$

$$24.2. z=ln\left(1+x+y^{2}+z^{2}\right); $$

$$M\_{1}\left(1;-1;1\right); M\_{2}\left(3;-5;1\right);$$

$$24.3. arctg\left(\frac{1,97}{1,02}-1\right);$$

$$24.4. \left\{\begin{array}{c}x^{2}+y^{2}=4y\\x^{2}+y^{2}=z^{2}\end{array}\right.; M\_{0}\left(-\sqrt{3}; 3;2\sqrt{3}\right);$$

$$24.5. z=3x^{3}+3y^{3}-9xy+10;$$

$$24.6. z=xy^{2}; при x+2y-1=0;$$

**Вариант 25\***

$$25.1. z=arc\sin(\left(xy\right)); $$

$$ \frac{x}{y}∙z\_{xx}^{''}+\frac{y}{x}∙z\_{yy}^{''}+\frac{2}{y}∙z\_{x}^{'}=2∙z\_{xy}^{''};$$

$$25.2. z=x^{yz}; M\_{1}\left(3;1;4\right); M\_{2}\left(1;-1;2\right);$$

$$25.3. ln\left(0,98^{4}+0,07^{3}\right);$$

$$25.4. \left\{\begin{array}{c}x^{2}+y^{2}=8\sqrt{2}x\\z=x^{2}+y^{2}-64\end{array}\right.; M\_{0}\left(2\sqrt{2};2\sqrt{6};-32\right);$$

$$25.5. z=xy\left(12-x-y\right);$$

$$25.6. z=xy; при x^{2}+y^{2}=1;$$

**Вариант 26\***

$$26.1. z=\cos(xy)+\sqrt{xy}; x^{2}∙z\_{xx}^{''}-y^{2}∙z\_{yy}^{''}=0; $$

$$26.2. z=\frac{x}{y}-\frac{y}{z}-\frac{z}{x}; M\_{1}\left(2;1;-1\right); M\_{2}\left(3;-1;2\right);$$

$$26.3. \frac{1,03^{2}}{\sqrt[3]{0,98∙\sqrt[4]{1,05^{3}}}};$$

$$26.4. \left\{\begin{array}{c}x^{2}+y^{2}+z^{2}=4\\x^{2}+y^{2}=z^{2}\end{array}\right.; M\_{0}\left(-1;1;-\sqrt{2}\right);$$

$$26.5. z=y\sqrt{x}-y^{2}-x+6y+5;$$

$$26.6. z=x^{2}-y^{2}+xy-x-y+4; при $$

$$ x+y-3=0;$$

**Вариант 27\***

$$27.1. z=\sin(\left(x+\sqrt{y}\right)); z\_{x}^{'}∙z\_{xy}^{''}-z\_{y}^{'}∙z\_{xx}^{''}=0;$$

$$27.2. z=ln\left(2x-3y^{2}+z\right);$$

$$ M\_{1}\left(3;-1;-2\right); M\_{2}\left(5;1;0\right);$$

$$27.3. \sqrt{3e^{0,07}+0,95^{2}};$$

$$27.4. \left\{\begin{array}{c}x^{2}+y^{2}=2y\\z=2,25-x^{2}\end{array}\right.; M\_{0}\left(0;1;2,25\right);$$

$$27.5. z=xy\left(6-x-y\right);$$

$$27.6. z=xy^{2}; при x+2y-1=0;$$

**Вариант 28\***

$$28.1. z=e^{-\cos(\left(x+2y\right))}; 4z\_{xx}^{''}=z\_{yy}^{''};$$

$$28.2. z=x^{y}-3xyz; M\_{1}\left(2;2;-4\right); M\_{2}\left(1;0;-3\right); $$

$$28.3. arctg\left(1-\frac{1,02}{1,98}\right);$$

$$28.4. \left\{\begin{array}{c}x^{2}+y^{2}=8\\x^{2}+z^{2}=4\end{array}\right.; M\_{0}\left(\sqrt{3};-\sqrt{5};1\right);$$

$$28.5. u=z^{3}+y^{2}+12zy+2x+x^{2};$$

$$28.6. z=\frac{1}{x}+\frac{1}{y} при x+y-2=0$$

**Вариант 29\***

$$29.1. z=\sin(xy)+\sqrt{xy}; x^{2}∙z\_{xx}^{''}-y^{2}∙z\_{yy}^{''}=0;$$

$$29.2. z=e^{x-yz}; M\_{1}\left(1;0;3\right); M\_{2}\left(2;-4;5\right);$$

$$29.3. \sqrt{5e^{0,02}+2,03^{2}};$$

$$29.4. \left\{\begin{array}{c}5x^{2}+3y^{2}=10\\4y^{2}+3z^{2}=27\end{array}\right.; M\_{0}\left(\sqrt{2};0;3\right);$$

$$29.5. u=x^{2}+y^{2}-xy+z^{2}+x-2z;$$

$$29.6. z=e^{xy}; при x+y-1=0;$$

**Вариант 30\***

$$30.1. z=e^{xy}+\sqrt{xy}; x^{2}∙z\_{xx}^{''}-y^{2}∙z\_{yy}^{''}=0;$$

$$30.2. z=x^{2}\left(y^{2}+z\right); M\_{1}\left(4;1;-3\right); M\_{2}\left(2;-1;1\right);$$

$$30.3. ln\left(0,98^{4}+0,07^{3}\right);$$

$$30.4. \left\{\begin{array}{c}x^{2}+y^{2}=6\sqrt{2}x\\z=x^{2}+y^{2}-36\end{array}\right.; M\_{0}\left(4;2\sqrt{2};-12\right);$$

$$30.5. z=x\sqrt{y}-x^{2}-y+6x+3;$$

$$30.6. z=2x+y; при x^{2}+y^{2}=1;$$

**Вариант 31\***

$$31.1. z=x^{2}e^{\frac{y}{x}}; x^{2}z\_{xx}^{''}+2xyz\_{xy}^{''}+y^{2}z\_{yy}^{''}=0;$$

$$31.2. z=ln\left(x^{2}+2y^{2}-z\right); M\_{1}\left(1;-1;2\right); M\_{2}\left(3;2;-1\right);$$

$$31.3. \sqrt{1,04^{1,99}+ln1,03};$$

$$31.4. \left\{\begin{array}{c}25\left(x^{2}+y^{2}\right)=4z^{2}\\5\left(x^{2}+y^{2}\right)=2z\end{array}\right.; M\_{0}\left(\frac{\sqrt{3}}{2};-\frac{1}{2};\frac{5}{2}\right);$$

$$31.5. u=x^{2}+y^{3}+12xy+2z+z^{2};$$

$$31.6. z=\frac{1}{x}+\frac{1}{y} при x+y-4=0$$

**Вариант 32\***

$$32.1. z=\sin(2xy)+\sqrt{xy}; x^{2}∙z\_{xx}^{''}-y^{2}∙z\_{yy}^{''}=0;$$

$$32.2. z=ln\left(x^{2}+y^{3}+z\right);$$

$$ M\_{1}\left(-2;1;-4\right); M\_{2}\left(1;-1;-1\right);$$

$$32.3. 2,06^{3,03}+3,03^{2,06};$$

$$32.4. \left\{\begin{array}{c}x^{2}+y^{2}=1\\x^{2}+y^{2}=2z\end{array}\right.; M\_{0}\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2};\frac{1}{2}\right);$$

$$32.5. u=x^{2}+y^{2}+z^{2}+2x+4y-6z;$$

$$32.6. z=x-2y; при x^{2}+y^{2}=1;$$